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Excel Files for Teaching Two Dimensional Motions and their Curvature

By Pavlos MIhas
Democritus University of Threace

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I. Introduction

In physics education research, student understanding ontopics of introductory mechanics has been thoroughly studiedfor several decades (for example Clement 1982, Halloun and Hestenes 1985). This research, has demonstrated that many students retain fundamentalconceptual difficulties, even after instruction (Kim and Pak 2002).

Two dimensional motions are examined in secondary education (for example: Motion on a circle, parabolas, elliptic motions of planets, circular pendulums, conical pendulums).

Two dimensional motions is a subject which requires the understanding of the vector nature of velocity, acceleration and force (Mihas & Gemoysakakis 2002).

We need to present to the students general principles, which they will apply to any kind of two dimensional motions.

The principles that students need to learn are: A) the velocity as a vector tangent to the orbit. B) The acceleration (centripetal acceleration) in case of constant speed is perpendicular to the velocity vector. The size of the centripetal acceleration depends on the speed and the radius of curvature of the orbit. C) Newton's second law. D) In the case of a tangential component of the force they have to think of the total acceleration as composed of two components: tangential acceleration and centripetal acceleration. E) The direction of the force with the direction of velocity can have an acute, right or obtuse angle if the measure of the velocity increases, remains constant or decreases and so the kinetic energy increases, remains constant or decreases. F) The curve can be approximated at a certain point with a circle, which has radius the radius of curvature which is expressed as $\rho = \dfrac{\left(\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2\right)^{3/2}}{\left(\frac{dx}{ds}\right)^3 \frac{d^2y}{dx^2}}$. The radius of curvature can have values from zero to infinity (in case of a rectilinear path o if $\frac{d^2y}{dx^2} = 0$. We will see examples of these cases in the following.

In this paper we examine several curves with their properties which can help as examples to elucidate the concepts, and we end up with the relative motion of a sliding body on a rotating disk which can gives us some unexpected results.

II. Sinusoidal Curve

One curve that can be used to introduce students to the ideas of radius of curvature and the relation between angle of velocity and acceleration is the sinusoidal curve.

The length s on the sinusoidal y =B·sinkx is expressed by use of elliptic integrals of the second kind:

$$s = \int \sqrt{1 + B^2 k^2 \cos^2 kx} \cdot dx = \sqrt{1 + B^2 k^2}\, E(\alpha, x)$$

where $\alpha = \dfrac{B \cdot k}{\sqrt{1 + B^2 k^2}}$

Since elliptic integrals are calculated very fast, it is easy to find solutions of the coordinates as a function of s. This permits to construct simulations. If we assume a speed that is increasing, with an increase of the speed which is proportional to the time, we can use this example to show the relation between angle of velocity and acceleration.

Usually the students pick up from the simulation some facts but they do notgenerally have a coherent picture. So they do not realize that the radius of curvature is infinit in the points where the inflection changes sides (in the sinusoidal curve at the points where the sign of y changes from plus to minus and vice versa).

For example they pick up the information that for a constant speed the acceleration should be perpendicular to the velocity, but they generalize it to the linear parts of the curve where radius of curvature is ρ =∞ and so $a_{centripetal} = v^2/\rho = 0$/

Author: *Democritus University of Threace, Greece.*
e-mail: pmichas@eled.duth.gr

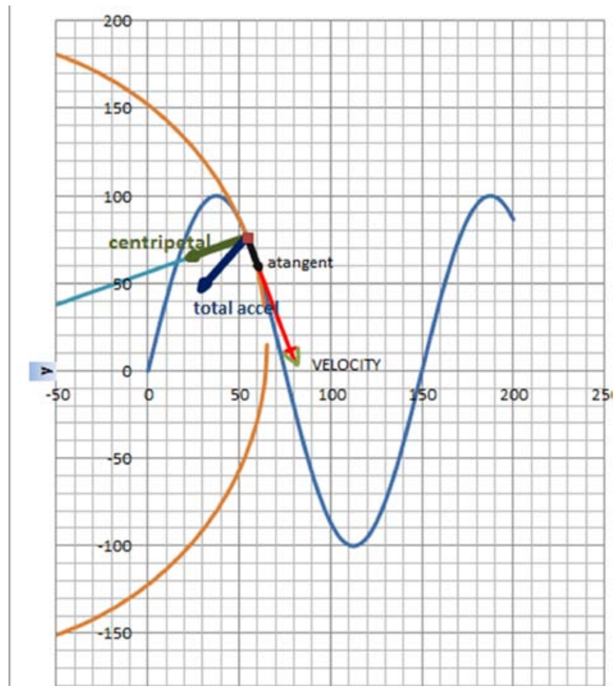

Figure 1: Sinusoidal with vectors of acceleration analyzed to entripetal and tangent

III. Tutorial Curve

This curve is used by McDermott etal in the Tutorials of introductory physics (in Greek p. 285 2011). This curve is a closed one and shows the change of direction of the centripetal acceleration but also the total acceleration the circle that corresponds to the curvature at the point where the moving body is located. The student can observe that when the body passes through points such as B and C the acceleration is almost tangent to the curve. This curve helps the student to understand that the angle of the acceleration to the velocity is acute when the speed increases and to see that the centripetal acceleration is greatest in the points where the radius of curvature is smallest. The worksheets comes in two forms (with macros, which permit a continuous change) and without, where the body moves by sliding a scroll bar (worksheet tutorial curve.xslx and tutorial curve macros). The points A,…E are points where the student should predict thedirection of the acceleration and also draw a circle with a radius equal to the radius of curvature.

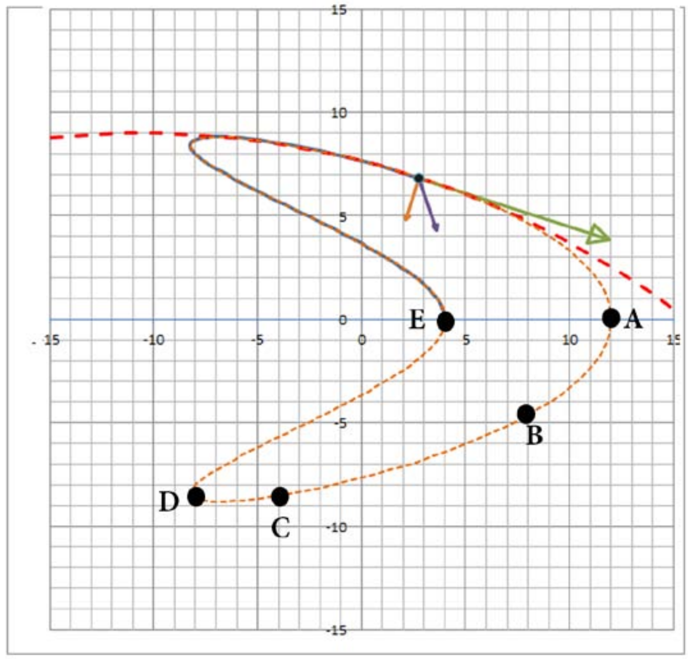

Figure 2: Closed curve used in Tutorials with constant tangential acceleration

IV. Neile Parabola

Aristotle was the first who was concerned with the composition of motions (Dugas p. 21). "Let a moving body be simultaneously be actuated by two motions that are such that the distances traveled in the same time are in a constant proportion. Then it will move into the diagonal of a parallelogram which has as sides two lines whose lengths are in this constant relation to each other" .Much later Galileo (Two New Sciences) analyzedthe motion of projectiles into two components and proved that the orbit described by the projectile will have the shape of a parabola.

Parabola was analyzed as performing on the same time an inertial motion and a motion under the influence of gravity. In Neile's parabola the motion is under the influence of two forces: A horizontal force arising from the reaction of the curve and a vertical force arising from the perpendicular component of the reaction and the weight. For a special value of the initial velocity, its vertical (perpendicular) component remains constant. In this case the total force has only a horizontal component. For larger speed the vertical motion will be an accelerated motion, and for lesser values of the speed it will be a decelerated motion. To find the curve for the special case of constant vertical component of the speed, let α be the initial horizontal component of the velocity and β the value of the vertical componentof velocity, then the vertical component of the reaction is equal to the weight for the case of constant β.

$\frac{dy}{dt} = \beta : \quad \frac{dx}{dt_0} = a$ .Fromwork-energytheorem:

$$\frac{1}{2} m \cdot v^2 = \frac{1}{2} m \cdot \left(\frac{dx}{dt} \right)^2 + \frac{1}{2} m \cdot \beta^2 - m \cdot g \cdot y \text{ and so } \frac{dx}{dt} = \frac{dx}{dy} \frac{dy}{dt} = \frac{dx}{dy} \cdot \beta = -\sqrt{a^2 - 2 \cdot g \cdot y} :$$

$$dx = -\frac{\sqrt{a^2 - 2 \cdot g \cdot y}}{\beta} dy \text{ and by integration we get: } x = \frac{(a^2 - 2gy)^{\frac{3}{2}}}{3g\beta} \quad y < y_{max} \text{ where } y_{max} = a^2/(2 \cdot g)$$

This curve was the first one to have its length calculated. This calculation is done easily by using the conservation of energy $\frac{ds}{dt} = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 - 2gy}$, then:

$$s = \int \sqrt{v_0^2 - 2gy} \cdot dt == \frac{1}{3g\beta} \left[v_0^3 - (v_0^2 - 2gy)^{3/2} \right]$$

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It is interesting that the radius of curvature of this curve is zero for $y=y_{max}$. If the speed is different from zero then the centripetal acceleration will be infinite. The radius of curvature can be expressed as:
which for Neile's curve is expressed as:

$$\rho = \frac{\sqrt{\alpha^2 - 2gy} \cdot \left(\alpha^2 + \beta^2 - 2gy\right)^{3/2}}{\beta^2 g}.$$

We can see that for the highest point $\rho=0$. In case of finite velocity we get infinite centripetal acceleration. The student can study the Neile's parabola with the spreadsheet "Neile curve"

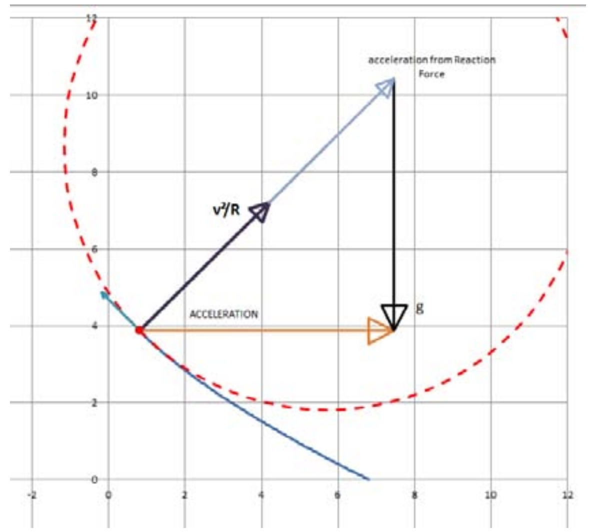

Figure 3: Neile's Parabola with circle of curvature and accelerations from spreadsheet "Neile Curve"

a) Cycloidalpendulum

In this pendulum the string is constantly in touch with a cycloid. For this pendulum the radius of curvature is studied easily, since it is equal to the length of the string that does not touch the upper curves. The bob of the pendulum describes cycloid. As the bob approaches the highest point the radius of curvature becomes smaller and eventually it becomes zero. In the case of vibration that starts at this point the velocity is zero and so the centripetal force is zero. This can be contrasted to what happens in Neile curve, At this point the total force (weight) is tangent to the curve.

The equations of the curve are: $x=R\cdot(\theta-\sin(\theta))$, $y=-R+R\cdot\cos(\theta)$ $(y<=0)$ where R is the radius of the circle that produces the cycloid. The length of the string for this pendulum is $L=4R$

We can find that the radius of curvature is:

$$\rho = 2\sqrt{-2yR}.$$

For $y=0$ (at the ends of the cycloid) the radius of curvature is zero. At the lowest point $y=-2R$ and then $\rho=4R=L$ = string's length.

As it was found in tests and exams the students could easily arrange the points 1,2,3 of figure according to their radii of curvature but.

The cycloidal pendulum has the property of a constant period $T = 2\pi\cdot\sqrt{\frac{l}{g}}$ that does not depend on the amplitude of the oscillations. This can be contrasted with the simple pendulum for which the period is $T = 4\cdot K(a)\cdot\sqrt{\frac{l}{g}}$ where $a=\sin(\varphi/2)$ (φ = amplitude) and K(a) is the complete elliptic integral.

The student can use the spreadsheet "cycloid pendulum" (with macros or not) to study the motion. The circle of curvature at the highest point has a minimum value which is zero if the bob starts from the cycloid. The student can compare the movement with the simple pendulum and see that the period of the cycloid pendulum is independent of the amplitude of the oscillation.

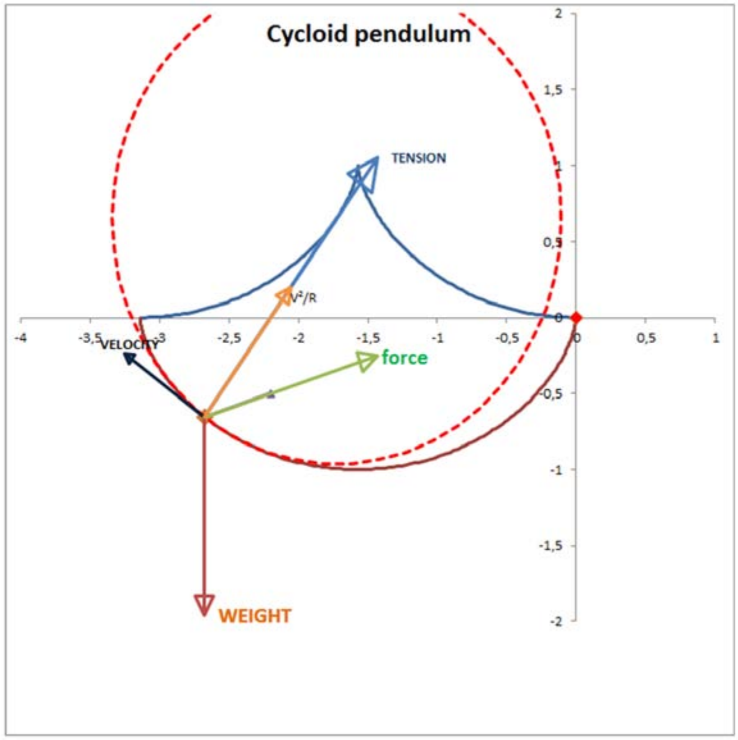

Figure 4: Cycloidal pendulum with curvatur circle and forces acting on the bob

b) *Motion of a charged body in the combination of a central force field and a magnetic field – or motion in a rotating frame, Foucault's pendulum*

In this case the magnetic force $\vec{F} = q\vec{v} \times \vec{B} = -q\vec{B} \times \vec{v}$ is the cause of a curving of the orbit. . With k/m = λ, ε =q.B/m, The equations of motion are

$$m\frac{d^2x}{dt^2} = k{\cdot}x + qBv_y \; m\frac{d^2y}{dt^2} = k{\cdot}y - qBv_x$$

Here k can be either positive or negative.

We consider a charged body starting with an initial velocity $v=v_{x0}$ and located at $x_0=0$, $y_0=R$. With $\varepsilon=qB/m$ and $\lambda=k/m$ we have two cases: $-\varepsilon^2+4\lambda>0$ and $-\varepsilon^2+4\lambda<0$ (which holds for a small repulsive force and always for attractive central force)

In this case we have an oscillatory motion.

c) *Solution for the equations of the motion of a charged body in the combination of a central force field and a magnetic field*

The equations: $\frac{d^2x}{dt^2} = \lambda{\cdot}x + \varepsilon{\cdot}v_y$, $\frac{d^2y}{dt^2} = \lambda{\cdot}y - \varepsilon{\cdot}v_x$ can be solved with elementary functions.

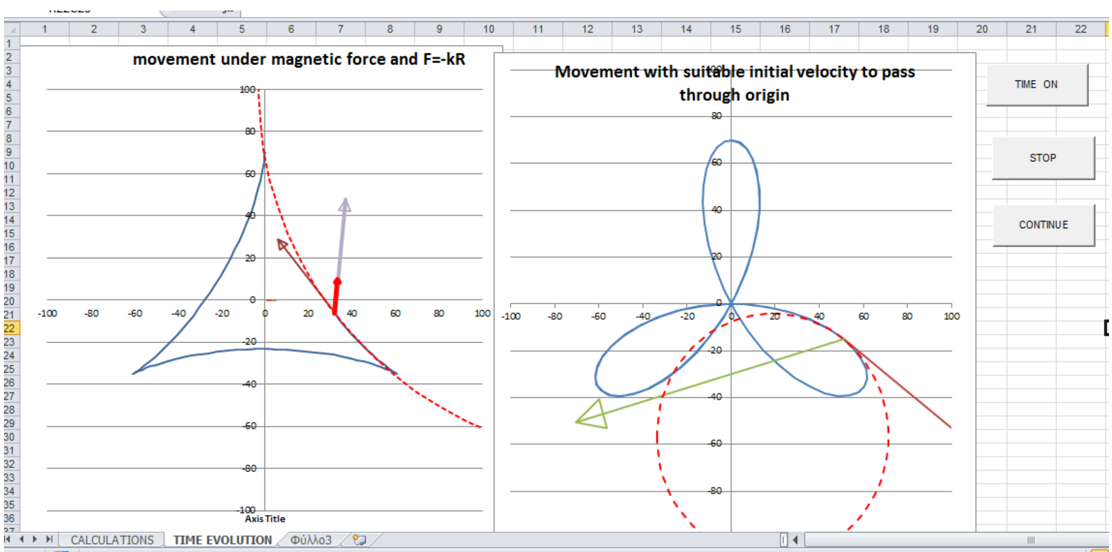

Figure 5: Movement in a combination of E and B, left with intial speed = zero, on the right with a sutibable initial horizontal speed so to pass through the origin (in the worksheet kappa= -50, B=5, mass=1)

By multiplying the second by $i=\sqrt{-1}$ and adding to the first we get the complex variable z=x+iy and the equation of motion: $\frac{d^2 z}{dt^2}=\lambda \cdot z - i\varepsilon \frac{dz}{dt}$ With $z = A \exp(\mu \cdot t)$

$$\mu^2 + i\varepsilon\mu - \lambda = 0 \quad \mu_{1,2} = \frac{-i\varepsilon \pm \mathrm{A}}{2}$$

If $-\varepsilon^2+4\lambda>0$ ($\lambda>0$ repulsiveforce) μ_1 and μ_2 are complex,

z=$f_1$exp((-ie+A)t/2)+$f_2$exp((-iε-A)t/2) where $f_1$and f_2 complex numbers. With initial conditions x=0, y=R and $(v_{x0} - \varepsilon R/2)2/A=\Delta$ we get:

f_1= M exp(iφ)

f_2= - Mexp(-iφ)

where $M = \sqrt{\Delta^2 + R^2}/2$ και $\phi = \arctan(R/\Delta)$ and Γ=A/2

$$z = M[\exp(i\phi - i\varepsilon t/2 + \Gamma t) - \exp(-i\phi - i\varepsilon t/2 - \Gamma t)]$$

$$x = M[\cos(\phi - \varepsilon t/2)\cdot\exp(\Gamma t) - \cos(\phi + \varepsilon t/2)\cdot\exp(-\Gamma t)]$$

$$y = M[\sin(\phi - \varepsilon t/2)\cdot\exp(\Gamma t) + \sin(\phi + \varepsilon t/2)\cdot\exp(-\Gamma t)]$$

If $-\varepsilon^2+4\lambda<0$ ($\lambda<0$ orsmall repulsive) we have $\sqrt{4\lambda - \varepsilon^2} = D$

we have two imaginary roots $\mu_1\mu_2$

$$\mu_1 = -(\varepsilon + D)/2,\ \mu_2 = -(\varepsilon - D)/2$$

$$z = -i\frac{(v_{x0} + \mu_2 \cdot R)\exp(i\mu_1 t) - (v_{x0} + \mu_1 \cdot R)\exp(i\mu_2 t)}{\mu_1 - \mu_2}$$

$$x = \frac{(v_{x0} + \mu_2 \cdot R)\sin(\mu_1 t) - (v_{x0} + \mu_1 \cdot R)\sin(\mu_2 t)}{\mu_1 - \mu_2}$$

$$y = -\frac{(v_{x0} + \mu_2 \cdot R)\cos(\mu_1 t) - (v_{x0} + \mu_1 \cdot R)\cos(\mu_2 t)}{\mu_1 - \mu_2}$$

If v_{x0}-ε·R/2=0,nd the body will pass through the origin Or x=R·sin(ε·t)·cos(D·t), y= R·cos(ε·t)·cos(D·t).

We can see that the body will pass through the origin with a period 2π/D.

With a suitable combination of E and B fields we can have the charge move with very small radius of curvature around a circle or much larger radius.

d) Similarity with Foucault's pendulum

This motion is similar to the motion of a Foucault's pendulum.

Actually the equations of motion are very similar. In Foucault's pendulum we have the Coriolis force $\vec{F}_{Coriolis} = -2m\,\vec{\omega} \times \vec{v}$ $\vec{\omega}$ *angular velocity* which has the same form as the magnetic force. This was mentioned in many papers (Semon & Schmieg 1981, Sivardiere 1983, Opat 1990) By comparing the two forces we can see that the Magnetic Force corresponds to an angular velocity ω=qB/(2m) =ε/2

We propose the following question: for what combination of initial speed and magnetic field will the orbit pass through the origin? For finding the answer we can use the similarity with Foucault's pendulum. Here we describe a pendulum located on a turntable. The bob of the pendulum describes a rotating ellipse. In the laboratory frame the bob has an initial velocity ($\vec{v} = \vec{\omega} \times \vec{R}$).

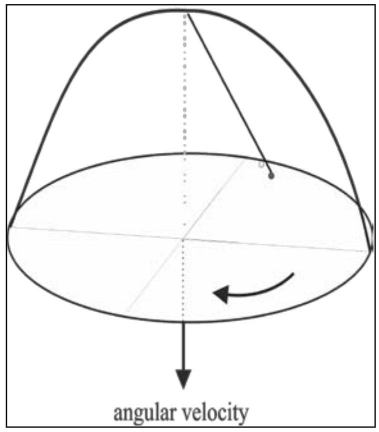

Figure 7: Foucault's pendulum

If on the other hand the pendulum is hanged from a basis in the laboratory frame then the motion starts with initial velocity equal to zero in the laboratory frame.

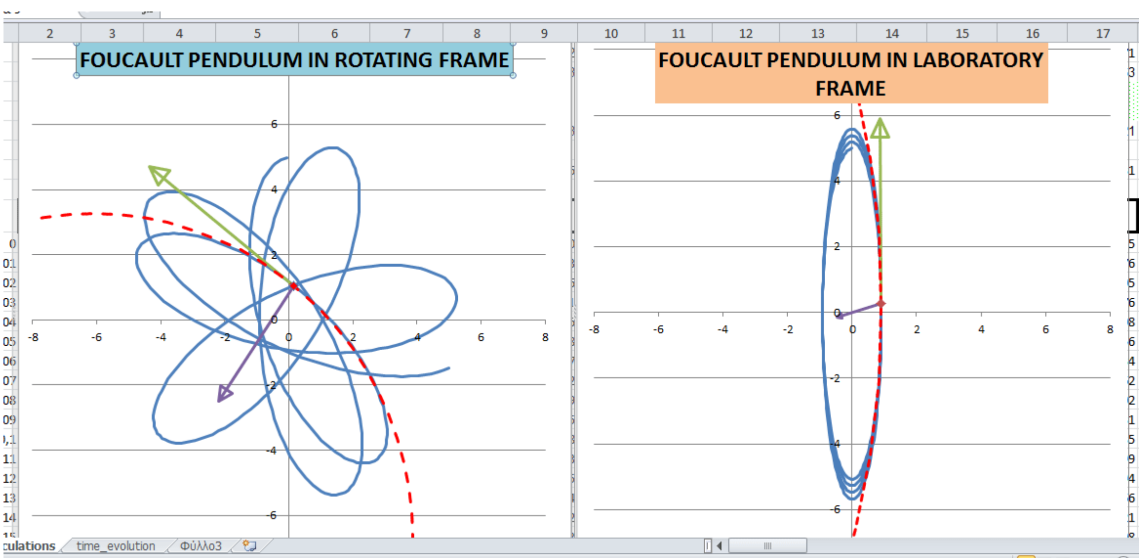

Figure 6: The Foucault pendulum in two frames of reference

The corresponding velocity in the rotating frame will be $\vec{v} = -\vec{\omega} \times \vec{R}$. The motion in the laboratory frame will be confined in a plane. In this case the pendulum will pass through the origin. The same thing can happen in a magnetic field if v_{x0}= qB/(4m). Then the motion will be that of a pendulum starting with $v_{x\ initial}$=0 in the laboratory frame. This can help us to calculate the time needed for a charge to pass through the origin.. For a body moving under a central attractive force F=-k·x in the laboratory frame $T = 2\pi\sqrt{m/k}$. In the case of a rotating frame then we should take into account the centrifugal force $m\omega^2 r$. The constant of the oscillatory motion is $-k_{effective} = -k + m\cdot\omega^2$. This will be the effective constant. Then for the magnetic field $-k_{effective} = -k + m\cdot(qB/(2m))^2 = -k + \varepsilon^2/4$. With $\lambda = -k_{effective}/m$ and λ_0 the corresponding value for the laboratory frame, $\lambda_0 = \lambda - \varepsilon^2/4$ and so the period is:

$$T = 4\pi\sqrt{m/(4\lambda - \varepsilon^2)}$$

V. Two Dimensional Motion on a Rotating Frame

A rotating frame is very interesting from the point of didactics of physics. There is a need for introduction of frames of reference and "inertial" forces. As Galili & Kaplan (1997) pointed out, standard Introductory Physics Courses do not usually consider more than one observer. He examined the presentation of Energy and Momentum in different frames of references.

Another issue that is usually neglected is the mass of the base. Galili & Kaplan examined the case of a ball sliding on base of finite mass and considers the energy and momentum in the corresponding frame of reference.

These issues will be considered here for the case of a rotating frame of reference. First will be addressed the rotating frame of reference with constant angular speed ω. In this case the inertia of the base can be considered infinite. Then there will be a consideration of a base of finite mass.

a) Base with constant angular velocity

Usually textbooks deal with relative motion in rotating frames of reference in connection with the Centrifugal force and the Coriolis force.

b) Absence of friction

The current paper deals with a particle lying at a point with coordinates (0, A). The rotating frame has an angular velocityω.

One characteristic of the rotating frame is that while the angular velocity is the same for every point of the rotating frame, the velocity each point is different.

c) Considerations of friction

Considering the effects of friction we obtain some results, which are not anticipated by the students (or even from physics teachers) if they do not grasp the notion of relative velocity. For a given location of the sliding body, the relative velocity has to be composed from the velocity of the point of the turntable and the velocity of sliding body.

Now the frictional force can be expressed as:

$$F_x = -\mu \frac{V_{x,relative}}{|V_{relative}|} \cdot m.g, \quad F_x = -\mu \frac{V_{y,relative}}{|V_{relative}|} \cdot m.g$$

This expresses the fact that the frictional force is directed opposite to the direction of relative velocity of the sliding body to the base.

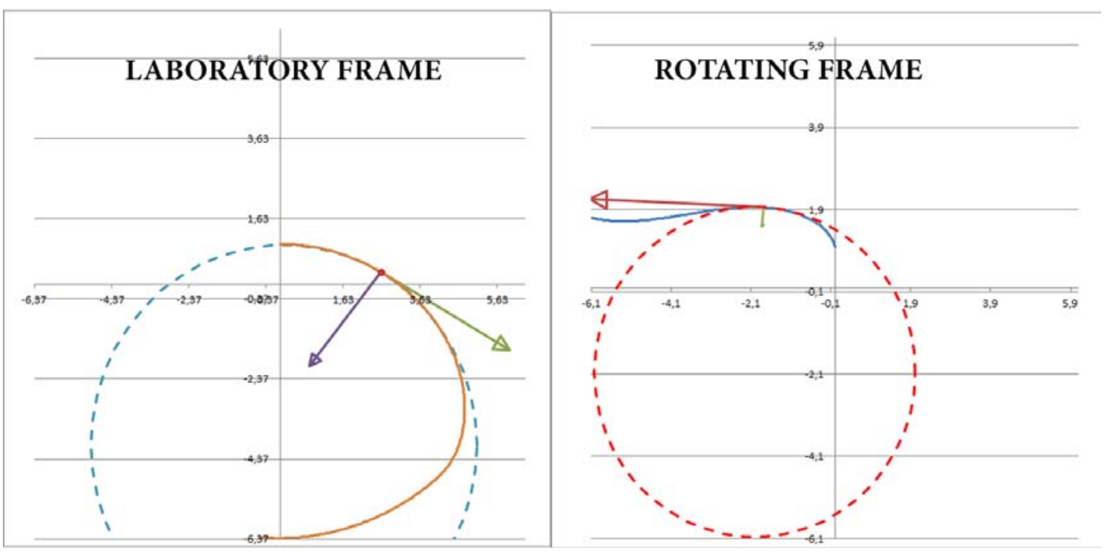

Figure 7: Otbits in different frames of reference

Since the direction of the frictional force is opposite to the direction of the relative velocity, the friction can act as an accelerating agent in the lab frame.

The path of a puck sliding on a turntable will become a curve. This can be explained by picturing the friction as dragging the sliding body from its rectilinear motion to a curved one. In this case the force vector and the velocity vector in the lab frame will have an acute angle. In the rotating frame the angle will be 180°.

Diagrams of the position of the body, its velocity and acceleration can be generating by using a spreadsheet or a Visual Basic program. In both cases are possible simulations of the motion.

The sliding body will get energy from the rotating frame. So the energy of the body will increase.

d) Energy considerations

The kinetic energy of the body in the laboratory frame will be constant in the case of no friction. The kinetic energy will increase in the case of frictional forces if the angular velocity of the base is constant.

A question that may be asked is: ‘In the lab frame the friction can act as an accelerating force, what is its role in the rotating frame?’ In the rotating frame the direction of the relative velocity is tangential to the path, so the direction of the friction is opposing the direction of velocity. So in the rotating frame the friction will decelerate the movement as it is anticipated from ‘normal’ cases. In the case of the observer on the rotating frame then a «centrifugal» potential can be used, to explain the energy increase as due to the action of the «centrifugal» force on the sliding body. This explanation holds true only if the angular velocity of the base is constant.

On the other hand if the rotating frame is of a finite mass it is expected that the energy

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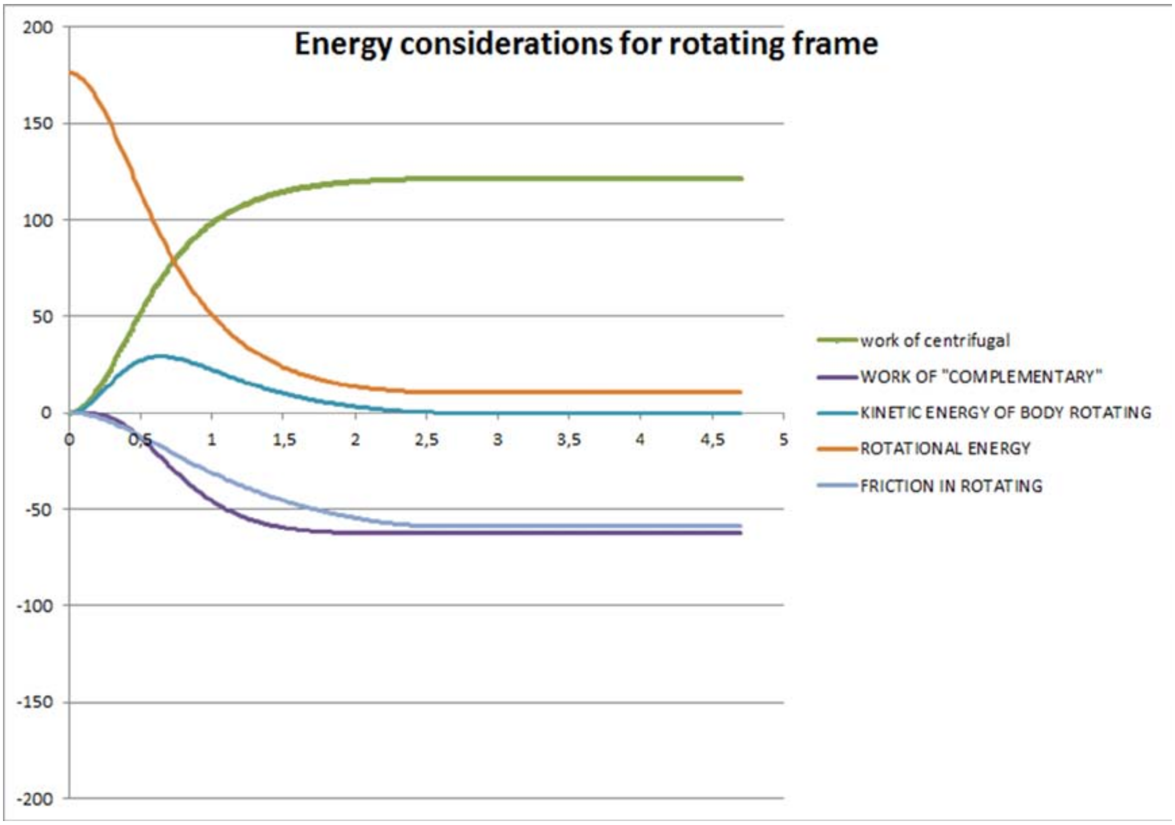

Figure 8: Energyin a rotating frame which is affected by the friction

of the rotating disk will decrease, due to the interaction of the sliding body (puck) with the rotating body. The sliding body acts through its frictional force (which is the one part of the action – reaction pair). The moment of this force decreases the angular velocity of the base. The decrease of the angular velocity can be calculated by the use of the conservation of angular momentum. The force vector and the velocity vector in the beginning of the motion will make a 90° angle in the laboratory, later on the angle becomes acute, and later it becomes obtuse. In this case the speed in the laboratory frame is reduced. When the body stops in the rotating frame, the angle becomes 90° in the lab frame.

In the case changing angular velocity the work of the friction plus the work of the centrifugal force in the rotating frame of reference will not in general be equal to the change of the kinetic energy. If the angular velocity of the base is affected because of its finite inertia, then the complementary acceleration must be taken care of, which gives an additional inertial force. The Coriolis force of course does not contribute to the change of the kinetic energy. The need to introduce an additional inertial force can be shown through a computer program, which shows to the user the sum of different terms if it is equal to the change of the kinetic energy in the rotating frame.

The final state of the body for the rotating observer will be that the sliding body will be motionless. For the observer in the lab frame the whole system will turn with a reduced angular speed. The body will be will describe a circle with the reduced angular speed of the base.

In case of rapid reduction of the angular speed, the graph of velocities in the rotating frame of reference ceases to have the periodic character, which has in the case where the base is not affected by the sliding body.

VI. Applications in Class

The teaching of two dimensional motions was done in the elementary education department of Democritus University (Mihas P. and Gemoysakakis T. 2007) and there was a special laboratory on friction in which except the basic ideas of friction the lesson was extended for the sliding friction on a rotating disk (Evangelopoulou and Mihas 2011). The results of this work were applied with less success to High School students (Evangelopoulou A and Mihas P.2012).

As was seen by Mihas & Gemoysakakis, the students who attended the classes could draw correctly the vectors of acceleration in different paths, and also draw the "circles of curvature", while for students who did not attend the classes they did not have any success.

The use of the files on friction for a "puck" moving on a rotating disk can be: a) Explanation of the direction of Friction in a turntable. b) Elucidation of the meaning of relative velocity in the lab frame and in the rotating frame of reference. c) The consideration of the direction of acceleration and the direction of the velocity vectors for the case of increasing, decreasing or constant speed. d) The application of the Work – Energy theorem in different cases. e) The need to introduce the complementary acceleration in the case of accelerating frame of reference.

A note on the software used in this paper

For each part of this paper there are Excel files used to draw the figures.

Section	Software
Sinusoidal Curve	sinusoidal MAKRO or sinusoidal (without a macro)
Tutorial Curve	tutorial curve
Neile Curve	Neile curve
Cycloidal pendulum	cycloid pendulum with macros
Motion of a charged body in a combination of electric field and magnetic field	Movement in combination of fields macros or without macros
Foucault's Pendulum	FOUCAULT PENDULUM MACROS
Two dimensional motion with friction	sliding on rotating frame small inertia plux mR2 sliding on rotating frame BIG INERTIA

All the files are found at http://kyriakosxolio.gr/w_dm_curv.html

References Références Referencias

1. E. Kim and S-J Pak (2002) Students do not overcome conceptual difficulties after solving 1000 traditional problems Am. J. Phys.70(7) pp. 759-765
2. Dugas Rene *A History of Mechanics* Editions du Griffon, Neuchatel, Switzerland, 1955 Edition
3. Evangelopoulou A. and Mihas P. (2011) The instruction of Galilean relativity and relative rotating motion to students of a Department of Elementary Education *Proceedings of the 6th Panhellenic Conference on Science Teaching and ICT in Education* (PP 326-334). Florina, University of West Macedonia, Greece. The paper is in Greek in the proceedings but there is an English translation in Research Gate.
4. Evangelopoulou A. and Mihas P. (2012) Perceptions of students of first grade of Greek High School about the friction, its laws and its role in the relative translational and rotational motion. A Proposal for a teaching intervention in a cooperative constructive-stic learning environment. *Themes of Sciences and Technology in Education 5 (1-2), pp. 5-26 (In Greek, An English Translation can be downloaded from Researchgate).*
5. Galili I, Kaplan D (1997) *Extending the application of the relativity principle: Some pedagogical advantages*, American Journal of Physics, pp 323 - 335, April 1997.
6. Geoffrey I. Opat.(1990) *Coriolis and magnetic forces: The gyrocompass and magnetic compass as analogs*, American Journal of Physics 68(12), 1173-1176.
7. I. A. Halloun and D. Hestenes, "The initial knowledge state of college physics students," Am. J. Phys. 53, 1043 (1985).
8. J Sivardiere (1983) *On the analogy between inertial and electromagnetic forces*, European Journal of Physics pp 162-164.
9. J.Clement, "Students' preconceptions in introductory mechanics, Am.J.Phys. 50, Jan .(1982).
10. Marc Semon, Glenn Schmieg, (1981) *Note on the analogy between Inertial and electromagnetic forces*, American Journal of Physics, 49(7) pp 689-690.
11. McDermott, L. C., & Shaffer, P. S. (1998). Tutorials of Introductory Physics. (Greek translation by Pavlos Mihas (2011). Typothito Publications).
12. Mihas P. and Gemoysakakis T, (2007) *Difficulties that students face with two-dimensional motion* Physics Education 42 (2) 163-169.